FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper E – Marking Guide

1. $= \int (\operatorname{cosec}^2 2x - 1)\ dx$ M1 A1

$= -\frac{1}{2}\cot 2x - x + c$ M1 A1 **(4)**

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2. *(a)* $-4\sin x + (2\cos y)\frac{dy}{dx} = 0$ M1 A2

$\frac{dy}{dx} = \frac{4\sin x}{2\cos y} = \frac{2\sin x}{\cos y} = 2\sin x \sec y$ M1 A1

(b) grad $= 2 \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 2$ B1

$\therefore\ y - \frac{\pi}{6} = 2(x - \frac{\pi}{3})$ M1

$6y - \pi = 12x - 4\pi$

$4x - 2y = \pi$ A1 **(8)**

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3. *(a)* $\frac{2+20x}{1+2x-8x^2} = \frac{2+20x}{(1-2x)(1+4x)} \equiv \frac{A}{1-2x} + \frac{B}{1+4x}$ B1

$2 + 20x \equiv A(1 + 4x) + B(1 - 2x)$ M1

$x = \frac{1}{2} \quad \Rightarrow\ 12 = 3A \quad \Rightarrow\ A = 4$ A1

$x = -\frac{1}{4} \quad \Rightarrow\ -3 = \frac{3}{2}B \quad \Rightarrow\ B = -2 \qquad \frac{2+20x}{1+2x-8x^2} \equiv \frac{4}{1-2x} - \frac{2}{1+4x}$ A1

(b) $\frac{2+20x}{1+2x-8x^2} = 4(1-2x)^{-1} - 2(1+4x)^{-1}$

$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-2x)^3 + \ldots$ M1

$= 1 + 2x + 4x^2 + 8x^3 + \ldots$ A1

$(1+4x)^{-1} = 1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(4x)^3 + \ldots$

$= 1 - 4x + 16x^2 - 64x^3 + \ldots$ A1

$\frac{2+20x}{1+2x-8x^2} = 4(1 + 2x + 4x^2 + 8x^3 + \ldots) - 2(1 - 4x + 16x^2 - 64x^3 + \ldots)$ M1

$= 2 + 16x - 16x^2 + 160x^3 + \ldots$ A1 **(9)**

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4. *(a)* $\overrightarrow{PQ} = (2\mathbf{i} - 9\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) = (3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ M1

$\therefore\ \mathbf{r} = (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ A1

(b) $6 + \mu = 2 \quad \therefore\ \mu = -4$ M1

$a + 4\mu = -9 \quad \therefore\ a = 7$ A1

$b - \mu = 1 \quad \therefore\ b = -3$ A1

(c) $= \cos^{-1}\left|\frac{3\times1 + (-1)\times4 + (-2)\times(-1)}{\sqrt{9+1+4}\times\sqrt{1+16+1}}\right|$ M1 A1

$= \cos^{-1}\frac{1}{\sqrt{14}\times\sqrt{18}} = 86.4°$ (1dp) M1 A1 **(9)**

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5. *(a)* $\int dy = \int -ke^{-0.2t}\ dt$ M1

$y = 5ke^{-0.2t} + c$ A1

$t = 0,\ y = 2\ \Rightarrow\ 2 = 5k + c, \quad c = 2 - 5k$ M1

$\therefore\ y = 5ke^{-0.2t} - 5k + 2$ A1

(b) $t = 2,\ y = 1.6\ \Rightarrow\ 1.6 = 5ke^{-0.4} - 5k + 2$ M1

$k = \frac{-0.4}{5e^{-0.4} - 5} = 0.2427$ (4sf) M1 A1

(c) as $t \to \infty,\ y \to h$ (in metres) M1

$\therefore$ "h" $= -5k + 2 = 0.787$ m $= 78.7$ cm $\therefore\ h = 79$ M1 A1 **(10)**

6.	*(a)*	$x = 0 \Rightarrow t^2 = 2$	
		$t \geq 0 \quad \therefore \; t = \sqrt{2} \qquad \therefore \; (0,\, 2 + \sqrt{2}\,)$	M1 A1
		$y = 0 \Rightarrow t(t + 1) = 0$	
		$t \geq 0 \quad \therefore \; t = 0 \qquad \therefore \; (2, 0)$	M1 A1
	(b)	$\dfrac{dx}{dt} = -2t$	M1
		area $= \int_{\sqrt{2}}^{0} t(t + 1) \times (-2t)\; dt$	A1
		$= \int_{0}^{\sqrt{2}} (2t^3 + 2t)\; dt$	
		$= [\,\tfrac{1}{2}t^4 + \tfrac{2}{3}t^3]_0^{\sqrt{2}}$	M1 A1
		$= (2 + \tfrac{4}{3}\sqrt{2}\,) - (0) = 2 + \tfrac{4}{3}\sqrt{2}$	M1 A1 **(10)**

7.	*(a)*	let $y = a^x$, $\therefore\; \ln y = x \ln a$	M1
		$\dfrac{1}{y}\dfrac{dy}{dx} = \ln a$	M1
		$\dfrac{dy}{dx} = y \ln a = a^x \ln a \;\; \therefore \;\; \dfrac{d}{dx}(a^x) = a^x \ln a$	A1
	(b)	$\dfrac{dy}{dx} = 4^x \ln 4 - 2^{x-1} \ln 2$	M1 A1
		$x = 0,\; y = \tfrac{3}{2},$ grad $= \ln 4 - \tfrac{1}{2}\ln 2 = \tfrac{3}{2}\ln 2$	M1
		$\therefore\;\; y = (\tfrac{3}{2}\ln 2)x + \tfrac{3}{2}, \quad 2y = 3x \ln 2 + 3, \quad 3x \ln 2 - 2y + 3 = 0$	M1 A1
	(c)	$4^x \ln 4 - 2^{x-1} \ln 2 = 0$	
		$(2^x)^2 \times 2 \ln 2 - \tfrac{1}{2}(2^x) \ln 2 = 0$	M1
		$\tfrac{1}{2}(2^x) \ln 2[4(2^x) - 1] = 0$	M1
		$2^x = \tfrac{1}{4},\; x = -2 \;\; \therefore\; (-2,\, \tfrac{15}{16})$	A2 **(12)**

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8. *(a)*

x	0	0.5	1	1.5	2	2.5	3
y	0	0.5774	0.7071	0.7746	0.8165	0.8452	0.8660

B2

(i) $\approx \tfrac{1}{2} \times 1 \times [0 + 0.8660 + 2(0.7071 + 0.8165)] = 1.96$ (3sf) — B1 M1 A1

(ii) $\approx \tfrac{1}{2}\times 0.5\times[0+0.8660+2(0.5774+0.7071+0.7746+0.8165+0.8452)]$

$= 2.08$ (3sf) — M1 A1

(b)

$= \pi \int_0^3 \dfrac{x}{x+1}\; dx$ — M1

$= \pi \int_0^3 \dfrac{x+1-1}{x+1}\; dx \;= \pi \int_0^3 (1 - \dfrac{1}{x+1})\; dx$ — M1

$= \pi[x - \ln|x + 1|\,]_0^3$ — M1 A1

$= \pi\{(3 - \ln 4) - (0)\} = \pi(3 - \ln 4)$ — M1 A1 **(13)**

Total **(75)**

Performance Record – C4 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	integration	differentiation	partial fractions, binomial series	vectors	differential equation	parametric equations	differentiation	trapezium rule, integration	
Marks	4	8	9	9	10	10	12	13	75
Student									